POWER AND TYPE I ERROR RATE COMPARISON OF MULTIVARIATE ANALYSIS OF VARIANCE

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Abstract: Four test statistic of multivariate analysis were compared when null hypothesis is true. The four compared test statistic were: Wilks' lambda, Pillai's trace, Lawley-Hotelling trace and Roy's largest Root. Data were simulated to compared the four test statistic under two different distributions (multivariate Gamma and multivariate normal), sample size (10, 20, 30, 40, 100, 200, 300, 400, 600, 700, 800 and 1000), number of variables (p = 2, 3, 4) and equal and unequal sample and variance co-variance matrix. The comparisons were done at two levels of significance (α = 0.01 and 0.05) using power of the test and type I error rate. The results showed that Roy's largest Root test statistic is better than all other test statistic considered when p = g = 2 because it has lowest power with type I error rate. But when p = g = 3 and p = g = 4, Wilks' lambda is the best, both when sample size are large and small.

Keywords: Type I error rate, normality, R statistical package

Introduction

Multivariate analysis of variance (MANOVA) can be viewed as a direct extension of the univariate (ANOVA) general linear model that is most appropriate for examining differences between groups on several variables simultaneously (Hair *et al*., 1987; Olejnik 2010). In ANOVA, differences among various group means on a single-response variable are studied. In MANOVA, the number of response variables is increased to two or more. The hypothesis concerns a comparison of vectors of group means. That is, Univariate analysis of variance is accomplished with a single test statistic, the F test. Multivariate analysis of variance utilizes four tests of statistical significance: (Wilks' lambda 1932, Pillai's 1959, the Lawley-Hotelling 1938, and Roy's 1945). Contrary to popular belief, they are not competing methods, but are complementary to one another. Rencher and Scott (1990) demonstrate that one avoids alpha inflation by following the procedure of reporting significant univariate effects only when the corresponding multivariate effect is significant. However, they show that the multivariate test can be considered to be significant if any of these four multivariate statistical tests is found to be significant.

There is recognition that MANOVA may not be the best choice in all cases in which multiple outcome variables are of interest. The choice of the analytic procedure does rest on several factors including the data, research design, and research questions. For example, if the outcome variables are uncorrelated or have high positive correlations, then MANOVA may not be as effective as conducting separate univariate ANOVAs (Tabachnick & Fidell, 2007). In contrast, MANOVA can have greater power compared to the univariate methods when there is a moderate to strong negative correlation between the dependent variables (Tabachnick & Fidell, 2007).

MANOVA has three basic assumptions that are fundamental to the statistical theory: independent, multivariate normality and equality of variance-covariance matrices. A statistical test procedure is said to be robust or insensitive if departures from these assumptions do not greatly affect the significance level or power of the test. But violations in assumptions of multivariate normality and homogeneity of covariance may affect the power of the test and type I error rate of multivariate analysis of variance (Johnson and Wichern, 2002; Finch, 2005a, 2008b; Fouladi and Yockey, 2002).

The purpose of this study is to identify the conditions under which each of the four is more robust when the assumption of normality and equality of variance co-variance matrix hold or violated using power and type I error rate. All these will be considered when null hypothesis (H_o) is true and when the number groups (g) and random variables (p) are two, three and four.

Materials and Methods

This work targeted at comparison of the four multivariate analysis of variance (MANOVA) test statistics which are Wilks' Lambda, Pillae's trace, Roy's largest root and Lawley's trace using R statistics.

1. Wilks' lambda

$$\Lambda = \frac{|E|}{|H+E|}$$

$$a = N - g - \frac{p-g+2}{2}$$

$$b = \begin{cases} \sqrt{\dfrac{p^2(g-1)-4}{p^2+(g-1)^2-5}}; & if\ p^2+(g-1)-5 > 0 \\ 1 & if\ p^2+(g-1)-5 \le 0 \end{cases}$$

and

$$c = \frac{p(g-1)-2}{2}$$

then

$$F = \left(\frac{1-\Lambda^{\frac{1}{b}}}{\Lambda^{\frac{1}{b}}}\right)\left(\frac{ab-c}{p(g-1)}\right) \sim F_{p(g-1),ab-c}$$

2. Hotelling – lawley Trace

$$T_o^2 = trace(\boldsymbol{HE^{-1}}) = \sum_{i=1}^{s} \lambda_i$$

Lets

$$s = \min(p, g-1)$$

$$t = \frac{|p-g-1|-1}{2}$$

and

$$U = \frac{N-g-p-1}{2}$$

then

$$F = \frac{2(su+1)}{S^2(2t+s+1)} T_o^2 \sim F_{s(2t+s+1),2(su+1)}$$

3. Pillai Trace

$$V = trace(\boldsymbol{H}(\boldsymbol{H}+\boldsymbol{E})^{-\boldsymbol{1}}) = \sum_{i=1}^{s}\left(\frac{\lambda_i}{1+\lambda_i}\right)$$

here

$$F = \left(\frac{2u+s+1}{2t+s+1}\right)\left(\frac{V}{s-V}\right) \sim F_{s(2t+s+1),s(2u+s+1)}$$

4. Roy's largest Root;

$$\theta = \left(\boldsymbol{HE^{-1}}\right) = \frac{\lambda_1}{1+\lambda_1}$$

$$F = \left(\frac{2U+2}{2t+2}\right)\phi_{max} \sim F_{(2t+2),(2u+2)}$$

Where $s = \min(p, g-1)$

$$\phi_i = \frac{\theta_i}{1-\theta_i} = \frac{1-\lambda_i}{\lambda_i}$$

$$\lambda_i = 1 - \theta_i$$

Methods

A simulation using R statistics was conducted in order to estimate the power of the test and Type I error rate for each of the previously discussed multivariate analysis of variance (MANOVA) test statistics (Wilks' Lambda, Pillae's trace, Roy's largest root and Lawley's trace) In each of the four different scenarios, that is ,when: null hypothesis is true, dataset normal or not ,equality of variance co-variance matrix hold or not. Three factors were varied in the simulation: number of groups (g), the number of variables (p) and significant levels (α).

Data generation

In each of the 1000 replications and for each of the factor combination, an n_1 x p data matrix X_1 , n_2 x p data matrix X_2 , n_3 x p data matrix X_3 and n_4 x p data matrix X_4 were generated using an R package for Multivariate Normal and Gamma. The programme also performs the (Box 1949) test for equality of covariance matrices by using the statistic:

$$M = c\sum_{i=1}^{k}(n_i - 1)\log\left|S_i^{-1}S_p\right| ,$$

Where $S_p = \frac{\sum_{i=1}^{k}(n_i-1)S_i}{n-k}$

and

$$c = 1 - \frac{2p^2+3p-1}{6(k-1)(p+1)}\left[\sum_{j=1}^{k}\frac{1}{n_j-1} - \frac{1}{n-k}\right]$$

$$\chi_B^2 = (1-C)M$$

And $S_i$and S_p are the $i-th$ unbiased covariance estimator and the pooled covariance matrix, respectively.

Box's M also has an asymptotic chi-square distribution with $\frac{1}{2}(p+1)(k-1)$ degree of freedom. Box's approximation seems to be good if each $n_i$exceeds 20 and if g and p do not exceed 5 (Mardia *et al.*, 1979);

H_o is rejected at the significance level α if $\chi_B^2 > \chi_{\alpha(v)}^2$

The following (in the table) are the levels used for each of the two factors

Multivariate distribution	Assumption of equality of variance co-variance matrix	Number of group(g) and Number of random variable(p)	Significant level(α)
Normal	Hold & violate	2	0.01, 0.05
	Hold & violate	3	0.01, 0.05
	Hold & violate	4	0.01, 0.05
Gamma	Hold & violate	2	0.01, 0.05
	Hold & violate	3	0.01, 0.05
	Hold & violate	4	0.01, 0.05

Results and Discussion

When data are multivariate normal with two groups (g) and two random variables (p), power of the test and type I error rate when null hypothesis is true for four test statistic are in this order : Wilks' Lambda ≥ Pillae's trace ≥ Lawley's trace ≥ Roy's largest root, at significant level 0.01 and 0.05 in the two scenarios (equal and unequal sample). And unequal variance co-variance matrix does not have much effect power and type I error rate when we have p=2 and g=2 (Tables 1 and 2).

For multivariate Gamma, the four test statistic are roughly equivalent and they are of this order Wilks' Lambda ≥ Pillae's trace ≥ Lawley's trace ≥ Roy's largest root, for both power and type I error rate irrespective of the significant level (0.01 or 0.05); when data are not normally distributed and variance co-variance matrix are unequal, it does not have much effect on power and type I error rate of the four test statistic (Tables 3 and 4).

When p and g = 3 and data are multivariate normal, type 1 error rate and power of the four statistic are in this order: Roy's largest root ≥ Lawley's trace ≥ Pillae's trace ≥ Wilks' Lambda. When sample size are very small, unequal variance co-variance matrix influence type I error rate of the test statistic (Tables 5 and 6).

For multivariate Gamma when p and g = 3, the four test statistic followed this order: Roy's largest root ≥Lawley's trace ≥ Pillae's trace ≥ Wilks'. Unequal variance co-variance matrix and unequal sample size affect the power and type I error rate of the four statistic (Tables 7 and 8).

Table 1: Multivariate Normal when variance co-variance matrix are equal

			Power of the Test for Multivariate Normal							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\boldsymbol{\Sigma_1 = \Sigma_2}$ **and p = g = 2**	$n_1 = n_2$	10,10	0.014	0.013	0.014	0.013	0.058	0.057	0.058	0.058
		100,100	0.104	0.103	0.104	0.101	0.438	0.436	0.438	0.422
		1000,1000	0.459	0.459	0.459	0.455	0.248	0.248	0.248	0.248
	$n_1 \neq n_2$	10,20	0.053	0.051	0.053	0.047	0.130	0.127	0.130	0.123
		100,200	0.065	0.064	0.065	0.064	0.853	0.851	0.853	0.825
		600,1000	0.264	0.263	0.264	0.262	0.567	0.567	0.567	0.564
			Type I error rate of the Test for Multivariate Normal							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\boldsymbol{\Sigma_1 = \Sigma_2}$ **and p = g = 2**	$n_1 = n_2$	10,10	0.017	0.013	0.017	0.000	0.047	0.043	0.047	0.000
		100,100	0.038	0.037	0.038	0.031	0.148	0.146	0.148	0.136
		1000,1000	0.590	0.589	0.590	0.588	0.800	0.800	0.800	0.799
	$n_1 \neq n_2$	10,20	0.018	0.015	0.018	0.000	0.066	0.062	0.066	0.029
		100,200	0.060	0.059	0.060	0.058	0.187	0.187	0.187	0.180
		600,1000	0.476	0.476	0.476	0.475	0.686	0.686	0.686	0.686

Table 2: Multivariate Normal when variance co-variance matrix are not equal

			Power of the Test for Multivariate Normal							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and $p = g = 2$	$n_1 = n_2$	10,10	0.018	0.017	0.018	0.018	0.175	0.167	0.175	0.152
		100,100	0.133	0.132	0.133	0.128	0.058	0.058	0.058	0.058
		1000,1000	0.087	0.087	0.087	0.087	0.503	0.503	0.503	0.501
	$n_1 \neq n_2$	10,20	0.042	0.040	0.042	0.039	0.068	0.067	0.068	0.068
		100,200	0.053	0.053	0.053	0.053	0.313	0.312	0.313	0.308
		600,1000	0.148	0.148	0.148	0.148	0.134	0.134	0.134	0.134
			Type I error rate of the Test for Multivariate Normal							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and $p = g = 2$	$n_1 = n_2$	10,10	0.013	0.011	0.013	0.000	0.064	0.058	0.064	0.013
		100,100	0.024	0.023	0.024	0.022	0.095	0.095	0.095	0.087
		1000,1000	0.262	0.262	0.262	0.260	0.503	0.503	0.503	0.502
	$n_1 \neq n_2$	10,20	0.005	0.004	0.005	0.000	0.030	0.026	0.030	0.011
		100,200	0.011	0.011	0.011	0.011	0.054	0.053	0.054	0.051
		600,1000	0.127	0.126	0.127	0.126	0.366	0.366	0.366	0.365

Table 3: Multivariate gamma when variance co-variance matrix are equal

			Power of the Test for Multivariate Gamma							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and $p = g = 2$	$n_1 = n_2$	10,10	0.025	0.024	0.025	0.024	0.065	0.065	0.066	0.065
		100,100	0.320	0.318	0.320	0.298	0.117	0.116	0.117	0.116
		1000,1000	0.569	0.569	0.569	0.564	0.816	0.816	0.816	0.812
	$n_1 \neq n_2$	10,20	0.010	0.010	0.010	0.010	0.080	0.078	0.080	0.079
		100,200	0.111	0.111	0.111	0.109	0.387	0.386	0.387	0.379
		600,1000	0.237	0.236	0.237	0.235	0.840	0.839	0.840	0.834
			Type I error rate of the Test for Multivariate Gamma							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and $p = g = 2$	$n_1 = n_2$	10,10	0.009	0.008	0.009	0.000	0.060	0.054	0.060	0.012
		100,100	0.064	0.062	0.064	0.057	0.169	0.169	0.169	0.162
		1000,1000	0.828	0.828	0.828	0.826	0.942	0.942	0.942	0.942
	$n_1 \neq n_2$	10,20	0.020	0.017	0.020	0.001	0.089	0.086	0.089	0.055
		100,200	0.094	0.094	0.094	0.089	0.282	0.282	0.282	0.277
		600,1000	0.713	0.713	0.713	0.710	0.895	0.895	0.895	0.892

Table 4: Multivariate gamma when variance co –variance matrix are not equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and $p = g = 2$	$n_1 = n_2$	10,10	0.043	0.040	0.043	0.037	0.072	0.070	0.072	0.071
		100,100	0.111	0.111	0.111	0.108	0.112	0.111	0.112	0.111
		1000,1000	0.891	0.891	0.891	0.885	0.975	0.975	0.975	0.973
	$n_1 \neq n_2$	10,20	0.161	0.154	0.161	0.116	0.086	0.084	0.086	0.084
		100,200	0.101	0.101	0.101	0.099	0.121	0.121	0.121	0.120
		600,1000	0.394	0.394	0.394	0.390	0.840	0.840	0.840	0.835
			Type I error rate of the Test for Multivariate Gamma							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and $p = g = 2$	$n_1 = n_2$	10,10	0.019	0.015	0.019	0.000	0.075	0.070	0.075	0.018
		100,100	0.074	0.072	0.074	0.063	0.218	0.215	0.218	0.205
		1000,1000	0.932	0.932	0.932	0.932	0.988	0.988	0.988	0.988
	$n_1 \neq n_2$	10,20	0.039	0.039	0.039	0.010	0.149	0.143	0.149	0.092
		100,200	0.172	0.172	0.172	0.161	0.359	0.357	0.359	0.351
		600,1000	0.853	0.853	0.853	0.852	0.951	0.951	0.951	0.951

Table 5: Multivariate normal when variance co-variance matrix are equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and $p = g = 3$	$n_1 = n_2$	10,10,10	0.013	0.016	0.017	0.051	0.054	0.060	0.061	0.093
		100,100,100	0.021	0.028	0.028	0.152	0.081	0.100	0.100	0.325
		1000,1000,1000	0.056	0.094	0.093	0.651	0.134	0.185	0.185	0.578
	$n_1 \neq n_2$	10,20,30	0.012	0.017	0.017	0.062	0.085	0.152	0.154	0.434
		100,200,300	0.017	0.026	0.026	0.084	0.204	0.407	0.399	0.959
		600,800,1000	0.037	0.067	0.068	0.267	0.109	0.160	0.160	0.392
			Type I error rate of the Test for Multivariate Normal							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and $p = g = 3$	$n_1 = n_2$	10,10,10	0.001	0.021	0.014	0.014	0.005	0.068	0.061	0.150
		100,100,100	0.005	0.036	0.037	0.138	0.026	0.117	0.113	0.401
		1000,1000,1000	0.239	0.623	0.624	0.844	0.481	0.834	0.834	0.967
	$n_1 \neq n_2$	10,20,30	0.000	0.031	0.025	0.074	0.003	0.086	0.079	0.284
		100,200,300	0.000	0.078	0.078	0.255	0.005	0.219	0.218	0.568
		600,800,1000	0.066	0.475	0.477	0.749	0.225	0.713	0.715	0.936

Table 6: Multivariate normal when variance co-variance matrix are not equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 3	$n_1 = n_2$	10,10,10	0.013	0.018	0.018	0.060	0.076	0.104	0.102	0.305
		100,100,100	0.020	0.026	0.026	0.141	0.060	0.065	0.065	0.126
		1000,1000,1000	0.016	0.020	0.020	0.063	0.059	0.063	0.063	0.122
	$n_1 \neq n_2$	10,20,30	0.013	0.018	0.018	0.071	0.066	0.093	0.092	0.293
		100,200,300	0.013	0.017	0.017	0.063	0.072	0.099	0.099	0.329
		600,800,1000	0.013	0.016	0.016	0.059	0.067	0.080	0.080	0.213
			Type I error rate of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 3	$n_1 = n_2$	10,10,10	0.006	0.063	0.035	0.060	0.034	0.137	0.110	0.272
		100,100,100	0.005	0.038	0.038	0.173	0.021	0.101	0.094	0.403
		1000,1000,1000	0.009	0.089	0.088	0.305	0.052	0.168	0.168	0.493
	$n_1 \neq n_2$	10,20,30	0.002	0.044	0.030	0.123	0.005	0.112	0.099	0.331
		100,200,300	0.000	0.036	0.033	0.159	0.006	0.095	0.093	0.385
		600,800,1000	0.003	0.059	0.059	0.264	0.013	0.141	0.141	0.456

Table 7: Multivariate gamma when variance co –variance matrix are equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and p = g = 3	$n_1 = n_2$	10,10,10	0.024	0.048	0.044	0.194	0.057	0.063	0.065	0.098
		100,100,100	0.035	0.055	0.054	0.375	0.109	0.145	0.146	0.345
		1000,1000,1000	0.232	0.416	0.411	0.994	0.140	0.196	0.196	0.738
	$n_1 \neq n_2$	10,20,30	0.014	0.021	0.022	0.057	0.062	0.081	0.081	0.218
		100,200,300	0.017	0.027	0.027	0.157	0.120	0.212	0.212	0.595
		600,800,1000	0.234	0.500	0.492	0.998	0.219	0.372	0.370	0.958
			Type I error rate of the Test for Multivariate Gamma							
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 = \Sigma_2$ and p = g = 3	$n_1 = n_2$	10,10,100	0.000	0.017	0.009	0.010	0.004	0.059	0.047	0.153
		100,100,100	0.007	0.079	0.078	0.292	0.044	0.222	0.220	0.606
		1000,1000,1000	0.911	0.990	0.990	1.000	0.974	0.997	0.997	1.000
	$n_1 \neq n_2$	10,20,30	0.001	0.034	0.030	0.097	0.000	0.114	0.106	0.379
		100,200,300	0.012	0.321	0.318	0.688	0.066	0.557	0.554	0.875
		600,800,1000	0.713	0.963	0.964	0.996	0.892	0.991	0.991	1.000

Table 8: Multivariate gamma when variance co-variance matrix are not equal

			Power of the Test for Multivariate Gamma							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 3	$n_1 = n_2$	10,10,10	0.016	0.024	0.026	0.071	0.078	0.106	0.111	0.259
		100,100,100	0.020	0.027	0.027	0.114	0.077	0.093	0.093	0.300
		1000,1000,1000	0.186	0.338	0.335	0.984	0.219	0..324	0.323	0.925
	$n_1 \neq n_2$	10,20,30	0.014	0.022	0.022	0.092	0.092	0.167	0.170	0.449
		100,200,300	0.022	0.039	0.038	0.272	0.107	0.181	0.180	0.624
		600,800,1000	0.099	0.218	0.216	0.926	0.356	0.587	0.583	0.996
			Type I error rate of the Test for Multivariate Gamma							
			α=0.01				**α=0.05**			
			Wilk	**Lawley**	**Pillai**	**Roy**	**Wilk**	**Lawley**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 3	$n_1 = n_2$	10,10,10	0.000	0.023	0.014	0.016	0.008	0.086	0.074	0.181
		100,100,100	0.009	0.110	0.108	0.382	0.044	0.259	0.255	0.649
		1000,1000,1000	0.982	1.000	1.000	1.000	0.999	1.000	1.000	1.000
	$n_1 \neq n_2$	10,20,30	0.001	0.071	0.067	0.160	0.007	0.181	0.171	0.417
		100,200,300	0.024	0.443	0.441	0.774	0.110	0.709	0.708	0.945
		600,800,1000	0.864	0.996	0.996	1.000	0.975	1.000	1.000	1.000

Table 9: Multivariate gamma when variance co –variance matrix are not equal

			Power of the Test for Multivariate Normal							
			α=0.01				**α=0.05**			
			Wilk	**Lawle**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.011	0.013	0.014	0.073	0.052	0.060	0.061	0.163
		100,100,100,100	0.015	0.022	0.022	0.230	0.073	0.103	0.103	0.663
		1000,1000,1000,1000	0.016	0.024	0.024	0.433	0.145	0.279	0.277	0.999
	$n_1 \neq n_2$	10,20,30,40	0.011	0.015	0.015	0.078	0.054	0.073	0.074	0.392
		100,200,300,400	0.014	0.028	0.028	0.351	0.059	0.085	0.056	0.434
		600,700,800,1000	0.029	0.079	0.078	0.970	0.102	0.200	0.199	0.987
			Type I error rate of the Test for Multivariate Normal							
			α=0.01				**α=0.05**			
			Wilk	**Lawle**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.000	0.015	0.013	0.103	0.000	0.058	0.044	0.515
		100,100,100,100	0.001	0.085	0.082	0.580	0.006	0.218	0.216	0.855
		1000,1000,1000,1000	0.455	0.978	0.978	1.000	0.712	0.997	0.996	1.000
	$n_1 \neq n_2$	10,20,30,40	0.000	0.064	0.058	0.334	0.000	0.134	0.133	0.702
		100,200,300,400	0.002	0.233	0.233	0.705	0.002	0.386	0.386	0.906
		600,700,800,1000	0.066	0.820	0.819	0.993	0.191	0.941	0.940	1.000

Table 10: Multivariate normal when variance co-variance matrix are not equal

			Power of the Test for Multivariate Normal							
			α=0.01				α=0.05			
			Wilk	Lawle	Pillai	Roy	Wilk	Lawle	Pillai	Roy
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.013	0.023	0.022	0.231	0.057	0.078	0.077	0.401
		100,100,100,100	0.016	0.026	0.026	0.479	0.066	0.087	0.087	0.591
		1000,1000,1000,1000	0.021	0.038	0.038	0.714	0.078	0.113	0.113	0.774
	$n_1 \neq n_2$	10,20,30,40	0.012	0.019	0.019	0.269	0.058	0.099	0.098	0.597
		100,200,300,400	0.012	0.017	0.017	0.164	0.055	0.069	0.069	0.245
		600,700,800,1000	0.019	0.039	0.039	0.519	0.068	0.096	0.096	0.637
			Type I error rate of the Test for Multivariate Normal							
			α=0.01				**α=0.05**			
			Wilk	**Lawle**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.000	0.016	0.008	0.120	0.000	0.076	0.054	0.520
		100,100,100,100	0.000	0.030	0.029	0.454	0.000	0.095	0.095	0.785
		1000,1000,1000,1000	0.001	0.423	0.421	0.971	0.036	0.723	0.723	0.997
	$n_1 \neq n_2$	10,20,30,40	0.000	0.005	0.003	0.185	0.000	0.024	0.015	0.516
		100,200,300,400	0.000	0.014	0.014	0.342	0.000	0.055	0.055	0.690
		600,700,800,1000	0.000	0.172	0.170	0.841	0.000	0.410	0.410	0.976

Table 11: Multivariate gamma when variance co –variance matrix are equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	Lawl	Pillai	Roy	Wilk	Lawle	Pillai	Roy
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.011	0.015	0.015	0.105	0.056	0.075	0.076	0.369
		100,100,100,100	0.013	0.018	0.018	0.142	0.063	0.079	0.079	0.382
		1000,1000,1000,1000	0.017	0.027	0.027	0.481	0.059	0.070	0.070	0.301
	$n_1 \neq n_2$	10,20,30,40	0.012	0.022	0.022	0.267	0.053	0.069	0.069	0.368
		100,200,300,400	0.013	0.020	0.020	0.310	0.061	0.093	0.093	0.487
		600,700,800,1000	0.013	0.018	0.018	0.226	0.066	0.093	0.093	0.465
			Type I error rate of the Test for Multivariate Gamma							
			α=0.01				**α=0.05**			
			Wilk	**Lawle**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.000	0.007	0.003	0.072	0.000	0.034	0.026	0.476
		100,100,100,1000	0.000	0.010	0.010	0.360	0.000	0.040	0.039	0.685
		1000,1000,1000,1000	0.000	0.033	0.033	0.510	0.000	0.109	0.109	0.783
	$n_1 \neq n_2$	10,20,30,40	0.000	0.020	0.012	0.328	0.000	0.089	0.084	0.727
		100,200,300,400	0.000	0.034	0.033	0.480	0.000	0.120	0.121	0.802
		600,700,800,1000	0.000	0.250	0.025	0.434	0.000	0.103	0.103	0.773

Table 12: Multivariate gamma when variance co –variance matrix are not equal

			Power of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	**Lawle**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.011	0.015	0.015	0.078	0.059	0.085	0.087	0.394
		100,100,100,100	0.014	0.019	0.019	0.187	0.079	0.119	0.118	0.813
		1000,1000,1000,1000	0.016	0.025	0.025	0.376	0.070	0.092	0.092	0.645
	$n_1 \neq n_2$	10,20,30,40	0.011	0.017	0.017	0.160	0.055	0.082	0.082	0.467
		100,200,300,400	0.014	0.027	0.027	0.379	0.069	0.129	0.129	0.883
		600,700,800,1000	0.012	0.015	0.015	0.150	0.067	0.095	0.095	0.534
			Type I error rate of the Test for Multivariate Gamma							
			α=0.01				α=0.05			
			Wilk	**Lawle**	**Pillai**	**Roy**	**Wilk**	**Lawle**	**Pillai**	**Roy**
$\Sigma_1 \neq \Sigma_2$ and p = g = 4	$n_1 = n_2$	10,10,10,10	0.000	0.023	0.012	0.117	0.000	0.075	0.056	0.485
		100,100,100,100	0.000	0.031	0.030	0.363	0.000	0.077	0.074	0.692
		1000,1000,1000,1000	0.000	0.085	0.081	0.595	0.004	0.137	0.137	0.827
	$n_1 \neq n_2$	10,20,30,40	0.000	0.131	0.104	0.550	0.000	0.259	0.238	0.856
		100,200,300,400	0.000	0.149	0.149	0.632	0.000	0.318	0.315	0.885
		600,700,800,1000	0.000	0.076	0.076	0.570	0.000	0.193	0.193	0.840

It is obvious that, when p and g = 4 and data are multivariate normal and gamma, Lawley's trace and Pillae's trace are roughly equivalent and the four statistic followed this order: Roy's largest root ≥ Lawley's trace ≥ Pillae's trace ≥ Wilks'. Therefore, when data are not normally distributed, unequal variance co-variance matrix and unequal sample does not have much effect on power and type I error rate (Tables 9 – 12).

In conclusion, it is obvious that when the assumption of equality of variance co –variance matrix is violated it affect the power and type I error rate of the four test of statistic when the sample size are very small (10:10, 10:20) most especially when p and g = 2. At p = g = 2 and p = g = 3, power and type I error rate are greatly affected when variance co-variance matrix are unequal and data are multivariate gamma, but when p and g = 4 unequal variance co-variance matrix and multivariate gamma does not have any effect on the four test statistic. Therefore, when p and g = 2 and null hypothesis is true, Roy's largest root is better than other three test of statistic while Wilks' Lambda is better than others when p = g = 3 and p = g = 4. From all tables above it is obvious that when level of significant (α) increases, power and type I error rate also increases.

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